

# Dark matter: A phenomenological existence proof

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**Abstract.** The non-Keplerian galactic rotational curves and the gravitational lensing data [1, 2, 3] strongly indicate a significant dark matter component in the universe. Moreover, these data can be combined to deduce the equation of state of dark matter [4]. Yet, the existence of dark matter has been challenged following the tradition of critical scientific spirit. In the process, the theory of general relativity itself has been questioned and various modified theories of gravitation have been proposed [5, 6, 7]. Within the framework of the Einsteinian general relativity, here I make the observation that if the universe is described by a spatially flat Friedmann-Robertson-Walker (FRW) cosmology with Einsteinian cosmological constant then the resulting cosmology predicts a significant dark matter component in the universe. The phenomenologically motivated existence proof refrains from invoking the data on galactic rotational curves and gravitational lensing, but uses as input the age of the universe as deciphered from studies on globular clusters.

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The abstracted result arises from the crucial observation that in general, without invoking physics of structure formation (i.e. primordial birth of galaxies and stars), it is impossible to constrain the ratio of the fractional matter and dark energy densities. However, an exception occurs, for the spatially flat FRW cosmology with Einsteinian cosmological constant. In the ‘matter-dominated’ epoch with cosmological constant, this exception completely determines the ratio of the matter and dark energy densities as a function of the age of the universe. Once the latter is brought in as an input, the said ratio is unambiguously fixed for the present epoch. It turns out that the resulting fractional density for matter  $\Omega_m$  is too large by a factor of 4 to 7 than the known  $\Omega_{sm} \approx 0.05$  associated with the standard model of particle physics. This circumstance

then unambiguously predicts a significant non-standard model component to  $\Omega_{\text{m}}$ . It is by rigorously establishing this outline in detail that the said claim is arrived at.

No uniqueness of the mentioned exception is implied; that is, there may exist other cosmologies where a similar circumstance occurs. I display the Newtonian constant  $G$  explicitly but set the speed of massless particles in vacuum  $c$  to be unity.

The setting of our argument is as follows. I consider the matter-dominated epoch with Einsteinian cosmological constant. By the latter one means that the dark energy equation of state  $w^\Lambda = p^\Lambda/\rho^\Lambda$  is confined to the choice  $w^\Lambda = -1$ . Then, a time independent  $\rho^\Lambda$  corresponds to the Einsteinian cosmological constant. Further, I consider the cosmology which is spatially flat. That is, spatial curvature constant  $k = 0$ . In the matter dominated epoch, I set the associated pressure to zero,  $p = 0$ . This is quite realistic. Thus, the FRW cosmology we consider is defined by the set:

$$\{k = 0, w^\Lambda = -1, \rho = \rho_{\text{m}}, p = p_{\text{m}} = 0, \rho^\Lambda = \text{constant}\} \quad (1)$$

Once the energy densities,  $\rho_{\text{m}}$  and  $\rho^\Lambda$ , and the pressures,  $p_{\text{m}}$  and  $p^\Lambda$ , associated with matter and cosmological constant  $\Lambda$  are specified the scale factor  $a(t)$ , parameterizing the cosmological expansion, is obtained by solving the Einstein field equations for a spatially flat FRW cosmology:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{m}} + \frac{8\pi G}{3}\rho^\Lambda \quad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_{\text{m}} - \frac{4\pi G}{3}(\rho^\Lambda + 3p^\Lambda). \quad (3)$$

In writing the above equation I set  $p_{\text{m}} = 0$  for the matter dominated epoch, and defined

$$\rho^\Lambda := \frac{\Lambda}{8\pi G}. \quad (4)$$

Equivalently, one can solve Eq. (2), and the equation for the conservation of energy-momentum (which follows from Eqs. (2) and (3))

$$\dot{\rho}_{\text{m}} + 3\left(\frac{\dot{a}}{a}\right)\rho_{\text{m}} + 3\left(\frac{\dot{a}}{a}\right)(\rho^\Lambda + p^\Lambda) = 0. \quad (5)$$

In obtaining the above equation  $\dot{\rho}^\Lambda$  was set to zero as required by the setting, and where in addition the last term on the left hand side now vanishes due to the fact that  $w^\Lambda = -1$

$(p^\Lambda = -\rho^\Lambda)$ , giving

$$\dot{\rho}_m + 3 \left( \frac{\dot{a}}{a} \right) \rho_m = 0. \quad (6)$$

The system of Eqs. (2) and (6) now forms a closed system. It governs the time evolution of  $a(t)$  and  $\rho(t)$ .

For the spatially flat FRW cosmology defined by the set (1) the division of Eq. (2) by the square of the Hubble parameter  $H = \dot{a}/a$  yields

$$1 = \Omega_m + \Omega_\Lambda \quad (7)$$

where

$$\Omega_m := \frac{8\pi G \rho_m}{3H^2}, \quad \Omega_\Lambda := \frac{8\pi G \rho^\Lambda}{3H^2} \stackrel{\text{Eq. (4)}}{=} \frac{\Lambda}{3H^2}. \quad (8)$$

Our task now is to obtain an expression for the time evolution of the ratio,  $\Omega_m : \Omega_\Lambda$ . Towards this end Eq. (6) gives the temporal dependence of the matter density on the cosmic scale factor

$$\rho_m(t) = \rho_m(t_\star) \left( \frac{a(t_\star)}{a(t)} \right)^3. \quad (9)$$

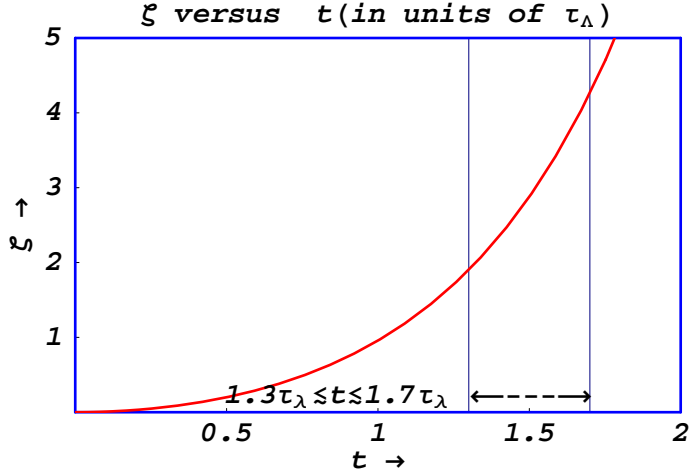
Here,  $t_\star$  marks the beginning of the matter dominated epoch. It is to be given as a physical input ( $t_\star \geq 10^6$  years, temperature  $T \approx 2000$  K).

The time dependence of the scale factor  $a(t)$  is now obtained by substituting for  $\rho_m(t)$  from Eq. (9) into Eq. (2), and solving the resulting differential equation for  $a(t)$ . This exercise yields the time evolution of the scale factor

$$a(t) = a(t_\star) \left[ \left( \frac{\rho_m(t_\star)}{\rho^\Lambda} \right)^{1/3} \sinh^{2/3} \left( \frac{\sqrt{3\Lambda}}{2} t \right) \right]. \quad (10)$$

This result agrees with that arrived at, e.g., by Frieman in Ref. [8, Eq. 3.6].<sup>‡</sup>

<sup>‡</sup> Padmanabhan [9, Eq. 27] and Sahni [10, Eq. 12] also note that scale factor  $a(t)$  is proportional to  $\sinh^{2/3} [(\sqrt{3\Lambda}/2)t]$ . However, as we next note, for the purposes of establishing the proposed thesis it is precisely the ‘proportionality constant’ that turns out to carry a pivotal importance.



**Figure 1.** The temporal evolution of  $\zeta := \Omega_\Lambda/\Omega_m$ . The label  $\leftrightarrow$  marks the present epoch with  $t_0 \approx (13.5 \pm 1.5) \times 10^9$  years in units of  $\tau_\Lambda$  ( $= 9 \times 10^9$  years, see text).

From a physical perspective, note that the nonlinearity of the gravitational field equations has the consequence that the “amplitude” of the scale factor carries a unique dependence on the indicated densities. Furthermore, Eq. (10) places a severe constraint on the relative initial densities by requiring the square bracket in Eq. (10) to take the value unity at  $t = t_*$ :

$$\frac{\rho_m(t_*)}{\rho^\Lambda} = \left[ \sinh \left( \frac{\sqrt{3}}{2} \frac{t_*}{\tau_\Lambda} \right) \right]^{-2} \quad (11)$$

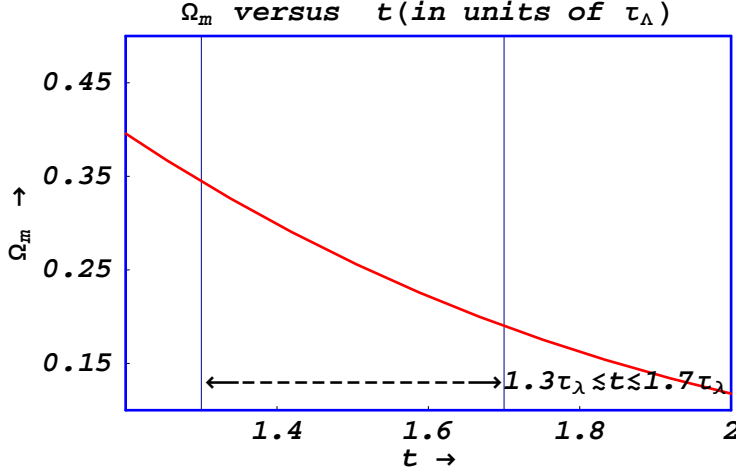
where we introduced  $\tau_\Lambda := \sqrt{1/\Lambda}$ . Substituting back  $a(t)$  from Eq. (10) in Eq. (9) provides the general time evolution of the relevant energy densities for  $t \geq t_*$

$$\frac{\rho_m(t)}{\rho^\Lambda} = \left[ \sinh \left( \frac{\sqrt{3}}{2} \frac{t}{\tau_\Lambda} \right) \right]^{-2}. \quad (12)$$

The result (11) also follows from the above equation for  $t = t_*$ . It reflects the internal consistency of the calculations. On dividing each of the densities in Eq. (12) by the critical density,  $\rho_c := 3H^2/(8\pi G)$ , the time evolution of the ratio  $\Omega_m : \Omega_\Lambda$  follows

$$\Omega_m(t) : \Omega_\Lambda(t) = 1 : \zeta(t) \quad (13)$$

where  $\zeta(t) := \sinh^2 \left( \sqrt{3} t / (2\tau_\Lambda) \right)$ . Once the age of the universe is specified by some independent observations, the considered cosmology uniquely determines the ratio



**Figure 2.** The temporal evolution of  $\Omega_m$ . The label  $\leftrightarrow$  marks the present epoch with  $t_0 \approx (13.5 \pm 1.5) \times 10^9$  years in units of  $\tau_\Lambda$  ( $= 9 \times 10^9$  years, see text).

$\Omega_m : \Omega_\Lambda$  and it in addition predicts the fractional matter density to be

$$\Omega_m(t) = (1 + \zeta(t))^{-1}. \quad (14)$$

This is the combined result of Eqs. (7) and (13).

Since 1998, there exists observational evidence for a positive cosmological constant [11, 12, 13, 14, 15]. The latest data and analysis of the luminosity data on Supernovae 1a, in addition, obtains [16] (for a  $k = 0$  cosmology)

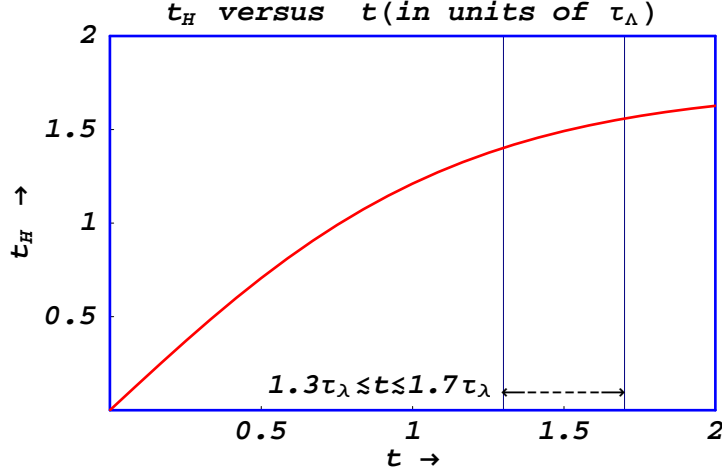
$$w^\Lambda = -1.023 \pm 0.090 \text{ (stat)} \pm 0.054 \text{ (syst)}. \quad (15)$$

These observations give us confidence that a spatially flat FRW cosmology with Einsteinian cosmological constant may indeed correspond to the physical reality. Taking  $\Lambda/(8\pi G) \approx 4 \times 10^{-47} \text{ GeV}^4$  yields  $\tau_\Lambda \approx 9 \times 10^9$  years. It sets the time scale for us. §

However, for the moment, one need not take any input from these data, except for working in units of  $\tau_\Lambda$  (for convenience). Instead, I use the present age of the universe  $t_0 \approx (13.5 \pm 1.5) \times 10^9$  years [19] as determined by the studies on globular clusters as the input. Thus the present epoch corresponds roughly to the range:  $1.3\tau_\Lambda \leq t \leq 1.7\tau_\Lambda$ . In this range, Figures 1 and 2 show that

$$1.9 \leq \zeta \leq 4.3, \quad 0.19 \leq \Omega_m \leq 0.35. \quad (16)$$

§ Indirect evidence for dark energy also comes from the data on the cosmic microwave background and the large scale structure observations [17, 18].



**Figure 3.** The range  $1.3\tau_\Lambda \leq t \leq 1.7\tau_\Lambda$  corresponds to  $1.4\tau_\Lambda \leq t_H \leq 1.6\tau_\Lambda$ . The  $t_H$  is defined as inverse of the Hubble parameter for spatially flat FRW cosmology with Einsteinian cosmological constant.

The latter may be compared with  $\Omega_m = 0.271 \pm 0.021(stat) \pm 0.007(syst)$  inferred by Astier et al. [16]. However,  $\Omega_{sm} \approx 0.05$  for the present epoch can only account for a small fraction, i.e., one fourth to one seventh of the predicted  $\Omega_m$  as given in Eq. (16). This state of affairs leaves the fractional density  $\Omega_m - \Omega_{sm}$  to point towards the existence of some form of non-standard model matter in the non-relativistic form. This, by definition, is the astrophysical/cosmic dark matter, with fractional density given by

$$\Omega_{dm}(t) = \Omega_m - \Omega_{sm} = (1 + \zeta(t))^{-1} - \Omega_{sm}(t). \quad (17)$$

For the present epoch, it yields roughly:  $0.14 \leq \Omega_{dm} \leq 0.30$ .

In the defined setting, the  $\Omega_{dm}$  as well as the  $\Omega_{sm}$  carry the same temporal evolution. It is immediately read off from Eqs. (9) and (10). In particular, the ratio  $\Omega_{dm} : \Omega_{sm}$  is frozen in time.

Before concluding it may be noted that the Hubble parameter as implied by the obtained scale factor given in Eq. (10) may also be evaluated. The result is,

$$H(t) : \frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3}} \coth \left( \frac{\sqrt{3}}{2} \frac{t}{\tau_\Lambda} \right). \quad (18)$$

It defines a characteristic time scale which is closely related to the age of the universe

$$t_H := \frac{1}{H(t)} = \sqrt{3} \tanh \left( \frac{\sqrt{3}}{2} \frac{t}{\tau_\Lambda} \right) \tau_\Lambda. \quad (19)$$

The range  $1.3\tau_\Lambda \leq t \leq 1.7\tau_\Lambda$  corresponds to  $1.4\tau_\Lambda \leq t_H \leq 1.6\tau_\Lambda$  (see Fig. 3). A more rigorous analysis along the lines of Ref. [20, Sec. 3.2] yields a very similar result.

Despite the on going debate on whether or not the general theory of relativity needs a modification while confronting the astrophysical and cosmological scene, the phenomenological existence proof given in the preceding discussion makes a strong case that such modifications are not required by the data on the galactic rotational curves or gravitational lensing. The dark matter required by these latter observations is the same dark matter that is here predicted within the general relativistic framework for the observationally favored spatially flat FRW cosmology with Einsteinian cosmological constant. The tantalizing possibility exists that the Pioneer anomaly may require a modification of the general theory of relativity in a subtle way [21, 22]. But that call comes not from the dark matter. The focus on new physics possibilities that dark matter offers perhaps ought to shift dramatically. One such direction is provided by the new and entirely unexpected theoretical discovery of a mass dimension one fermionic field of spin one half [23, 24]. That fermionic field, by virtue of its mass dimension, explains the darkness of dark matter as a natural consequence of the spacetime symmetries underlying the freely falling frames of general relativity.

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